



K23P 1409

Reg. No. :

Name :

III Semester M.Sc. Degree (C.B.S.S. – Reg./Supple./Imp.)
Examination, October 2023
(2020 Admission Onwards)
MATHEMATICS

MAT3C12 : Functional Analysis

Time : 3 Hours

Max. Marks : 80

PART – A

Answer **four** questions from this Part. **Each** question carries **4** marks.

1. State and prove Riesz lemma.
2. Show that c_{00} cannot be a Banach space with respect to any norm.
3. If a closed map F is bijective, then show that its inverse F^{-1} is also closed.
4. State open mapping theorem.
5. Let X be an inner product space and $x \in X$. Prove that $\langle x, y \rangle = 0$ for all $y \in X$ if and only if $x = 0$.
6. Let E be an orthogonal subset of an inner product space X and $0 \notin E$. Show that E is linearly independent.

PART – B

Answer **four** questions from this Part without omitting any Unit. **Each** question carries **16** marks.

Unit – I

7. a) Define a normed space and draw the sets $\{x \in \mathbb{R}^2; \|x\|_p = 1\}$ for $p = 1, 2$ and ∞ .
b) If X is a finite dimensional normed space then show that every closed and bounded subset of X is compact.

P.T.O.



8. a) Show that every linear map from a finite dimensional normed space is continuous.
- b) Let X and Y be normed spaces and $F : X \rightarrow Y$ be a linear map such that $R(F)$ of F is finite dimensional. Show that F is continuous if and only if the zero space $Z(F)$ is closed in X .
9. a) State and prove Hahn-Banach separation theorem.
- b) If X is a normed space and X' is strictly convex then show that for every subspace Y of X and every $g \in Y'$, there is a unique Hahn-Banach extension of g to X .

Unit – II

10. a) State and prove Uniform Boundedness Principle.
- b) Give the geometric interpretation of Uniform Boundedness Principle.
11. State and prove Closed Graph Theorem.
12. a) State and prove Bounded Inverse Theorem.
- b) Let X be a Banach space in the norm $\| \cdot \|$. Show that there is a norm $\| \cdot \|'$ on X which is comparable to the norm $\| \cdot \|$, but in which X is not complete.

Unit – III

13. a) State and prove Gram-Schmidt orthonormalization process.
- b) State and prove Riesz-Fischer theorem.
14. a) If H is a non-zero separable Hilbert space over K then show that H has a countable orthonormal basis.
- b) If E is a convex subset of an inner product space X , then show that there exists at most one best approximation from E to X .
15. a) State and prove Riesz representation theorem.
- b) Let H be a Hilbert space and for $f \in H'$, let y_f be the representer of f in H . Show that the map $T : H \rightarrow H'$ given by $T(f) = y_f$ is a surjective conjugate-linear isometry.
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